THE THEORY OF HADLEY's QUADRANT,

OR

THE RULES FOR THE CONSTRUCTION

AND

USE OF THAT INSTRUMENT

DEMONSTRATED.

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ADVERTISEMENT.

THE following ſheets are deſigned as a Supplement to the Directions for the Uſe of HADLEY's QUADRANT, &c. *That tract is intended to teach the practical part, this to ſhow the theory. Each part being thus printed ſeparately, the maker and practical obſerver will not be perplexed with abſtruſe demonſtrations in no wiſe neceſſary for them; nor the man of ſcience find the reaſoning interrupted and obſcured by matters foreign to the theory.*

THE THEORY OF HADLEY's QUADRANT, &c.

1. LET *NIn*, and *RHr*, fig. 1. repreſent the ſections of the plane of the figure by the ſurfaces of two ſpeculums ſtanding thereon. Let *BI* be a ray of light falling on a point *I* of the firſt ſpeculum, and reflected thence to a point *H* of the ſecond ſpeculum, and by it into the line *HO*; then the angle between the incident ray, and the ray after two reflections, is double the angle between the ſpeculums.

2. Since the incident ray *BI* is in the plane of the figure, the reflected ray will be ſo too *; and becauſe the ray *IH* is in that plane, ſo will alſo the reflected ray *HO* *; the rays *BI* and *HO* therefore being both in the plane of the figure, let them (when produced) meet in *E*, and let the two ſections *NIn* and *RHr* (produced) meet in *S*, then will *ISH* be the inclination of the reflecting planes to each other †; but *ISH* is the difference of the angles *IHR* and

* *Smith's* Optics, Art. 7. † Def. 6. and Prop. 19. El. XI.

HIS *. Produce *EH* backwards to *A*, and because by the laws of reflection † $IHR = EHS = RHA$ ‡, therefore *IHA* is double of *IHR*. In like manner because † $HIS = BIN = SIE$ ‡, therefore *HIE* is double of *HIS*; but *IEH* is the difference between *IHA* and *HIE* *, that is between twice *IHR* and twice *HIS*, or it is double the difference of the angles *IHR* and *HIS*, that is double the angle *ISH*.

3. This demonstration is general, but the figure represents the disposition of the glasses and course of the rays in the fore observation. It may be more convincing, and help us to clearer notions, to see it particularly applied to the back observation. Fig. 2 then represents the disposition of the glasses and course of the rays in the back observation. In this let the two sections *NIn* and *RHr*, as before, meet in *S*. Produce *HS* to *T*, and *IST* is the inclination of the reflecting planes to each other ‖. Let *BI* the incident ray, and *HO* the ray after two reflections (when produced) meet in *E*; and produce *OHE* to *A*, so that *IEA* may be

* 32 El. I. † Smith's Optics, Art. 9. ‡ 15 El. I.
‖ Def. 6. Prop. 19. El. XI.

the

the *obtuſe* angle which theſe rays make with each other. Then is *IEA* the ſum of *IHE* and *HIE* *, the former of which (by the laws of reflection †) is double of *IHS*, and the latter double of *HIS*. Therefore *IEA* is double the ſum of *IHS* and *HIS*; that is double of *IST* *.

4. Let *SH* and *SI*, fig. 3. repreſent the ſections of the plane of the figure by the ſurfaces of two ſpeculums ſtanding perpendicularly thereon. Let theſe ſections meet in *S*, and *ISH* will be the inclination of the reflecting planes, and *S* the point where their common ſection paſſes the plane of the figure, to which it is alſo perpendicular ‡. Let *Q* be an object placed between the ſpeculums, and in the plane of the figure. Several repreſentations of that object will be formed by the ſucceſſive reflections of theſe ſpeculums; all which will lie in a plane paſſing through the object, and perpendicular to both the reflecting planes; that is in the plane of the figure as was before ſhewn, par. 2. ‖ But theſe repreſentations

* 32 El. I. † *Smith's* Optics, Art. 9, &c. as before. ‡ Def. 6. and Prop. 19. El. XI. ‖ *Smith's* Optics, Art. 7, &c.

or images of the object Q, will alſo lie in the circumference of a circle drawn on that plane, whoſe center is S, and radius SQ the perpendicular diſtance of the object from the common ſection of the two ſpeculums.

5. Draw QT perpendicular to the ſpeculum SH, produce QT to A, ſo that $TA=TQ$, and Q being conſidered as the focus of rays incident on SH, A will be the focus of reflected rays *, or place of the image after one reflection from SH †. But the right-angled triangles QTS and ATS having the ſides TA and TQ equal, and ST common, will have their hypothenuſes SQ and SA equal ‡. Therefore the circle deſcribed with the center S and radius SQ, will paſs through A. — Moreover the arch HA is equal to HQ.

6. For the like reaſon if a be the image made by one reflection from the other ſpeculum SI, it will lie in the circumference of this circle paſſing through Q; and the arch Ia is equal to IQ.

7. The image a may be conſidered as an object in reſpect of the ſpeculum SH ||.

* *Smith*'s Optics, Art. 202. † Ibid. Art. 25.
‡ 4 El. 1. || *Smith*'s Optics, Art. 111.

Take

Take therefore the arch $HB = Ha$, and B will be the image of that image, or the image seen in the speculum SH by two reflections; first from SI, and then from SH *.

8. The image A may in like manner be considered as an object in respect of the speculum SI †. Take the arch $Ib = IA$, and b is the other image made by two reflections and seen in the speculum SI.

9. Again take $HC = Hb$, and C will be the image seen in the speculum SH after three reflections; first from SH, then from SI, and then from SH again.

10. Take $Ic = IB$, and c is the other image seen (in SI) after three reflections.

11. Again take $HD = Hc$, and D is the image seen in SH after four reflections.

Take $Id = IC$, and d is the other image seen after four reflections.

12. Again take $HE = Hd$, and E is the image seen in SH after five reflections; and so on.

13. *Cor.* 1. Since $Qa = 2Ia$, and $Ba = 2Ha$, subtract the former from the latter, and there

* Par. 5. † *Smith's* Optics, Art. 111.

remains

remains $\mathcal{Q}B = 2 \times \overline{Ha - Ia} = 2HI$. Again, $Bc = 2Ic$ and $Dc = 2Hc$; subtract the former from the latter, and there remains $BD = 2 \times \overline{Hc - Ic} = 2HI$, and so on. Make therefore the arches $\mathcal{Q}B$, BD, DF, &c. each equal to $2HI$; and B, D, F, &c. will be the successive images of the object $\mathcal{Q}$ after an even number of reflections.

14. In like manner take $\mathcal{Q}b$, bd, df, each equal to $2HI$; and b, d, f, &c. will be the place of the images seen in the other speculum after an even number of reflections.

15. *Cor.* 2. The distances of each of the images B, D, F, &c. or of b, d, f, &c. from the object $\mathcal{Q}$, measured along the arch of the circle; is equal to the arch HI multiplied by the number of reflections.

16. *Cor.* 3. Again; since $HA = H\mathcal{Q}$, and $HC = Hb$, take the former from the latter, and there remains $AC = \mathcal{Q}b = 2HI$ *. Therefore if $\mathcal{Q}A$ be taken equal to $2\mathcal{Q}H$, and the several arches AC, CE, &c. each equal to $2HI$; then A, C, E, &c. will be the place of the images seen in SH after an odd number of reflections.

* Par. 14.

17. In

17. In like manner take $Qa = 2QI$, and then the ſeveral arches ac, ce, &c. each equal to $2HI$ and a, c, e, will be the place of the images ſeen in the other ſpeculum after an odd number of reflections.

18. *Cor.* 4. The diſtance of the firſt image from the object or QA, is equal to $2QH$: that of the third image or QC ($= QA + AC$) is equal to $2QH + 2HI$; that of the fifth image (or $QA + AE$) is $2QH + 4HI$, &c. the diſtance of each image from the object Q, meaſured along the arch of the circle, being equal to the arch HI multiplied by the number of reflections ſave one, and increaſed by twice QH the diſtance of the object from the ſpeculum.

19. The ſame is true of a, c, e, &c. the images ſeen in the other ſpeculum after an odd number of reflections.

20. *Cor.* 5. If the two ſpeculums, keeping their inclination to each other, be turned round their common ſection as an axis; the images B, D, F, &c. formed by an even number of reflections will be immoveable; becauſe the intercepted arch HI is not altered: but the alternate images A, C, E, &c. formed by an odd number of reflections,

will

will change their places by double the change of the arch QH, or double the angular motion of the ſpeculums.

21. *Cor.* 6. Hence (in fig. 1) with the center S and radius SB deſcribe a circle cutting in A, the laſt reflected ray HO produced backwards; and A will be the place of the image of the object B after both reflections. For all the images of this object lie in the circumference of that circle *, and the laſt image muſt lie in the direction of the laſt reflected ray †; therefore it is in A their common interſection.

22. Let an object be placed at C in the reflected ray OH produced, ſo as to be ſeen by direct viſion through the ſpeculum H along with the reflected image A. The image A will continue immoveable notwithſtanding the two ſpeculums be turned together round their common interſection as an axis ‡; and therefore will ſeem always to adhere to the object C.

23. Let the eye be placed at E the interſection of the incident ray with the ray after two reflections; then BEA, or BEC

* Par. 4 and following. † *Smith's* Optics, Art. 101, 102. ‡ Par. 20.

the

the angle which these two rays make with each other, is also the angle which the object *B* makes with it's image *A*: it is also the angle which the two objects *B* and *C* subtend to the naked eye at *E*. This angle *BEA* has been shown to be equal to twice *ISH* *.

24. If the eye be placed any where else in *HE* produced as at *O*, the doubly reflected image *A* and the object *C* will still appear to coincide as before: but the angle which the objects *B* and *C* subtend to the naked eye is now *BOC*, differing from *BEA* by the angle *EBO*, or *IBO* †.

25. Let now the speculum *NIn* be turned about the point *I*, so that the doubly reflected image of the object *C* (placed in the line *HO* produced) may be made to *coincide* with that object itself seen directly by the eye at *O*; then is the angle *ICH* double of *ISH* the angle between the speculums. For in this case the incident ray is *CI*, and the ray after two reflections is *HO*, which produced backwards passes through *C* by the supposition of the coincidence aforesaid. Therefore the inter-

* Par. 2. † 32 El. I.

section of these two rays is at *C* the object itself, and consequently *ICH* the angle of their intersection is double of *ISH* the angle of the speculums *.

26. The intersection before marked *E*, in this case removes to *C* on the other side of *H*; and *S*, the point of intersection of the two sections *NIn* and *RHr*, falls on the other side of the line *IH* †.

27. If

* Par. 2.

† To consider the motion of the points *E* and *S* more particularly, imagine the speculum *NIn* to turn round the point *I*, while the other speculum *RHr*, and likewise the position of the ray *HO*, remains fixed.

And first let the speculum *NIn* turn so that the point *S* may continually recede from *H*. Then the point *E* will continually recede from *H* towards *O*. When the two speculums become parallel, the point *S* and likewise the point *E* will go off *in infinitum*, as is evident because *BEA* is twice *ISH*; therefore when the latter vanishes, so does the former. In this case the incident ray and ray after two reflections are parallel.

Continue the motion of the speculum *NIn*, and the sections *NIn* and *RHr* will now meet on the other side, in *SH* continued. The intersection of the incident ray and ray after two reflections will now also remove to the other side, in *OH* continued. This will appear, if instead of considering *HO* as the ray after two reflections, we consider it as the incident ray, and therefore *HI* as a ray given in position and incident on the second speculum *NIn*; *IC* will then be the ray after two reflections by *Smith*'s Optics, Art. 11.

Continue the motion of the speculum *NIn* and the point of intersection of these two rays will continue its motion from *A* towards *H*, and passing the point *H* will recede beyond *O in infinitum*. Thus these two rays will again be parallel. This happens when the speculum *NIn* makes a right angle with the speculum *RHr*.

Continue this motion, and the intersection of the incident ray and ray after two reflections will a second time remove to

27. If the object C be removed to ſuch a diſtance that the angle ICH vaniſhes, the angle ISH will alſo vaniſh, and the ſpeculums become parallel to each other.

The incident ray CI, and the ray after two reflections HO, in this caſe are alſo parallel, their interſection C being removed to an infinite diſtance.

28. *Scholium.* If when the index ſtands at the beginning of the diviſions, the two ſpeculums are put into a parallel poſition, by turning one of them about till the reflected image of ſome diſtant object C coincides with that object ſeen directly, the inſtrument is then ſaid to be *adjuſted*. The inſtrument being thus adjuſted and the ſpeculums fixed, the index will always ſhow

to the other ſide, in OH continued. When the face of the ſpeculum comes into the direction IH, the interſection of thoſe two rays will then be in H. After this the back of the ſpeculum will be turned towards RHr for an intire ſemi-revolution, till the face again comes into the direction IH, and the two rays aforeſaid again interſect in H; the ſpeculums then making an acute angle the ſame way as at firſt.

If the motion of the ſpeculums be ſtill continued, the interſection of the two rays at E will recede from H towards O *in infinitum*, till the ſpeculums are parallel as before.

The incident ray and ray after two reflections are twice parallel in one ſemi-revolution of the ſpeculum NIn; *viz.* when the two ſpeculums are parallel, and alſo when they are perpendicular to each other. In the other ſemi-revolution, the back of the ſpeculum NIn is turned towards the other ſpeculum RHr.

the double inclination of the ſpeculums, either on the quadrantal arch, or arch of exceſs, according as the point *S* falls on one ſide or the other of the perpendicular let fall from *I* on *RHr* produced.

If, when the ſpeculums are put into a parallel poſition as before, the index points to any number of degrees or minutes on the quadrantal arch (inſtead of 0) then it will always give the double inclination too much on the quadrantal arch, and equally too little on the arch of exceſs, by the number of degrees and minutes ſhown as before; and contrariwiſe.

29. The angle *BIC*, which any two objects *B* and *C* ſubtend at *I* the center of the index glaſs, is equal to the ſum of the two angles *BEC* and *ICE* *.

30. *Scholium* 2. The quadrant being adjuſted as before by a diſtant object, the former of theſe angles *BEC* will be ſhown on the quadrantal arch when the reflected image of *B* coincides with *C* ſeen directly. The latter of theſe angles *ICE* will be ſhown on the arch of exceſs when the reflected image of *C* coincides with that object itſelf ſeen

* 32 El. I.

directly,

directly. Or the ſum of theſe two angles will be ſhown at once on the quadrantal arch, if, when the object *C* and its image coincide, the index be made to point at the beginning of the diviſions; and then be carried forward till the reflected image of *B* coincides with *C* ſeen directly.

31. In figure 4. let *I* be the index-glaſs, ſtanding parallel to *H* the fore horizon glaſs; let *R* be the back horizon glaſs, ſtanding at right angles to *H* and *I*. Draw *PI* and *PR* perpendiculars to *I* and *R* meeting each other in *P*. Let *QH* be perpendicular to *H*. Let *BI* be the ray incident on *I*, and reflected from thence into *IH*, from thence into *HO*, the axis of viſion for the fore horizon glaſs. Let *IR* be the ray reflected from the index-glaſs to the back horizon glaſs, and from thence into *RY* the axis of viſion for that glaſs. Then by the laws of reflection $PIH = PIB$ *. Moreover $PIH = IHQ$ †. Therefore the angle which the line, joining the centers of the index-glaſs and fore horizon glaſs, makes with a perpendicular to the index-glaſs when the index ſtands at the beginning of the diviſions,

* *Smith's* Optics, Art. 8. † 29 El. I.

sions, is equal to the angle of incidence on the index-glass in that position; and this is the constant angle of incidence on the fore horizon glass.

32. That $PIR = PIH + HIR$ is self-evident, whatever be the position of the perpendicular PI. But PIH is the angle of reflection from the index-glass to the fore horizon glass, and PIR the angle of reflection to the back horizon glass, which are equal to their respective angles of incidence. Therefore the angle of incidence on the index-glass in the back observation, exceeds the angle of incidence in the fore observation (the perpendicular PI standing in the same place) by the angle HIR which the centers of the two lesser mirrors subtend at the center of the great one.

33. When I and H are parallel, PIH is equal to IHQ, and IPR is a right angle. Therefore PRI is the complement of PIR, or of $PIH+HIR$, that is of $IHQ+HIR$. Therefore the angle of incidence on the back horizon glass, is constantly the complement to 90 degrees, of the sum of the angle of incidence on the fore horizon glass, and the angle which the centers of the two lesser

leſſer mirrors ſubtend at the center of the great one.

34. Let *RN* repreſent the ſurface of the back horizon glaſs, *RY* the ray reflected from it or the axis of viſion, and produce *YR* to *M*. Let *OH* be the axis of viſion for the fore obſervation. Produce *OH* till it meets *RY* in *Y*, and let *OH* meet *IR* in *Z*. Then the angle *RYZ*, which the two axes of viſion make with each other, is the difference of *ZRM* and *YZR* *. Now the former of theſe $ZRM = 2 \times ZRN$ † $= 2 \times PIR$ ‡ $= 2 \times PIH + 2 \times HIZ$. The latter of thoſe angles or $YZR = ZHI + HIZ$ * $= BIH + HIZ$ ‡ $= 2 \times PIH + HIZ$ †. Subtract the latter from the former, and we have their difference, or *RYZ* equal to *HIZ*, or *HIR*. Therefore the angle which the two axes of viſion (for the fore and back obſervation) make with each other, is equal to that which the two leſſer mirrors ſubtend at the center of the great one.

35. To ſhow the correction neceſſary to be made in taking the height of a building by reflection from water, let figure 5 repreſent the poſition of the octant when the

* 32 El. I. † *Smith's* Optics, Art. 8. ‡ 29 El. I.

fore

fore obſervation is uſed; figures 6 and 7 the poſition of the octant in the back obſervation. In theſe figures let LR be the ſurface of the water. BL the building, AL its reflected image ſeen in the water. R the point of reflection from the water. I the point of reflection from the index-glaſs. H the point of reflection from the horizon-glaſs, through which the extremity of the image A is ſeen in the water. O the place of the eye. E the interſection of the ray (from B the top of the building) incident on the index-glaſs with that ray after two reflections. S the point where the ſections of the ſpeculums by the plane of the octant meet each other; and continue HS to T in figures 6 and 7. Then is AEB the angle ſhown on the limb, being double of ISH in figure 5, and double of IST in figures 6 and 7 *, and BRL is the angle of the elevation of the building above the level of the water, ſuppoſing the eye placed at R the point of reflection. Now $2 \times BRL = = BRA$ † $= AEB + EBR$ in figures 5 and 6, and equal to $AEB - EBR$ in figure 7 ‡; that is $2 \times BRL = AEB + IBR$ in figures 5

* Par. 1, &c. † *Smith's* Optics. Art. 8. ‡ 32 El. I.

and

and 6, and equal to *AEB*−*IBR* in figure 7. Estimate therefore the angle *IBR*, which the middle of the index-glass, and the point of reflection from the water, makes at the top of the building: add this to the angle shown on the limb in the fore observation; add or subtract it in the back observation, according as the ray incident on the water passes without or within the index-glass; and the sum or difference gives the angle *BRA*; half of which is the true angle of elevation.

36. To find the errors occasioned by a given deviation of the reflecting planes from their perpendicular position.

It may be convenient to consider the planes in question as the planes of several great circles of a sphere; and the point in which the two reflecting planes and plane of the octant all concur, as the center of that sphere. Their mutual inclinations may then be found by the common rules of spherical trigonometry.

37. In figure 8 let *C* be the center of the sphere, where the three planes before mentioned all meet. Let the plane of the great circle *TNEH* represent the plane of the

octant. Let the plane of the great circle *NCP* represent the plane of the index-glass; and the plane of the great circle *HCP*, the plane of the horizon-glass. Then is *NC* and *HC* the intersections of the two reflecting planes with the plane of the octant, and *PC* their common intersection with each other. Moreover the spherical angles *PNT* and *PHN* are the inclinations of the reflecting planes to the plane of the octant which we here suppose equal; the spherical angle *NPH* is their inclination to each other. The arch *NH* measures the angle *NCH*, which the two sections of the reflecting planes by the plane of the octant, make with each other; and which is half the angle shown by the index on the limb. With the pole *P* describe a great circle *LDEF*, cutting the circle *PND* at right angles in *D*; the circle *PFH* at right angles in *F*, and the circle *TNEH* obliquely in *E*; *CE* being the intersection of the plane of the oblique circle *LDEF* with the plane of the circle *TNEH*. Through *P* and *E* draw a great circle; this will meet the circle *LDEF* at right angles in *E*; and the plane of this circle will pass through both

both *PC* the common interſection of the reflecting planes, and alſo through *CE* the interſection of the plane of the oblique circle *LDEF* with the plane of the circle *TNEH*; the plane of this circle is ſeen edgewiſe in the figure. The ſpherical angle *PEN* which this circle makes with the circle *TNE* will be the inclination of *PC* the common interſection of the reflecting planes to the plane of the circle *TNEH*, that is to the plane of the octant, and *NED* will be the complement of this angle. Laſtly, let the radius *TC*, lying in the plane of the great circle *TNEH*, repreſent a line drawn on the plane of the octant parallel to the axis of the teleſcope; paſſing both through *CH* the common ſection of the plane of the octant and horizon-glaſs, and through *PC* the common ſection of the reflecting planes. Through *P* and *T* draw the great circle *PTL*, meeting the circle *TNE* obliquely in *T*, but the circle *LDEF* at right angles in *L*; then will *TCH* be the (obtuſe) angle which the line *TC* makes with the ſection *CH*. The arch *PT* is the meaſure of *PCT*, the angle which the line *TC* makes with *PC* the common ſection of the reflecting planes;

the arch *TL* is the complement of the arch *PT*, or it is the diſtance of the parallel circle from the great circle, mentioned by Hadley in his fifth corollary; that great circle being here repreſented by *LDEF* *.

38. Fig. 9 repreſents the ſtereographic projection of theſe ſeveral circles on the plane of the circle *TNEH* in figure 8, that is on the plane of the octant; the dotted lines *CT*, *CN*, *CH*, repreſent the ſeveral radii *CT*, *CN*, *CH*, lying in the plane of the circle *TNEH*. The ſeveral circles of the ſphere before mentioned are all repreſented by circles in figure 9, according to the laws of the ſtereographic projection.

39. Theſe things laid down, becauſe the ſpherical angles *PNT* and *PHN* are by ſuppoſition equal, the arches *EN* and *EH*, likewiſe *ED* and *EF* are reſpectively equal: whence *EN* is equal to $\frac{1}{2}NH$; therefore *EN* is the meaſure of $\frac{1}{2}$ of the angle ſhown by the index on the limb. The arch $ED = \frac{1}{2}DF$, therefore *ED* is the meaſure of half the ſpherical angle *NPH*, or of half the true inclination of the reflecting planes to each other, or it is the meaſure of $\frac{1}{2}$

* See Philoſ. Tranſ. Nr 420.

of

of the corrected angle. Now in the right-angled spherical triangle *NDE* we have the hypothenuse *EN*, and the angle *END* = = *PNT*, to find the base *ED*. Therefore as radius is to sine *END* (or cosine of deviation from the perpendicular); so is sine *EN* to sine *ED*, which multiplied by 4 gives the corrected angle.

40. Next to find the inclination of *PC* the common section of the reflecting planes to the plane of the octant, or to find *NED* the complement of that angle. In the right-angled triangle *NDE* we have the hypothenuse *EN* and the angle *END* as before, to find the other angle *NED*. Therefore as radius is to tangent *END*, so is cosine *EN* to cotangent *NED*, or the tangent of the inclination of the common section of the reflecting planes to the plane of the octant.

41. Next to find *PCT* the angle which a line drawn on the plane of the octant, parallel to the axis of the telescope and passing through the common section of the reflecting planes, makes with that common section; or rather to find *TL* the arch which measures the complement of that angle.

First

First we have $ET=TH-HE=TH-EN$. Then in the right-angled spherical triangle TLE we have the hypothenuse ET, the angle $TEL=NED$ (just found) to find the perpendicular TL. Therefore as radius is to sine ET, so is sine TEL to sine TL, or distance of the parallel from the great circle in Hadley's fifth corollary.

42. Lastly by that corollary, as radius is to the sine complement of that distance, so is the chord of double the true inclination of the reflecting planes, or chord of the corrected angle; to the chord of the true angle between the observed objects: that is radius is to the sine complement of the distance aforesaid as the sine of half the corrected angle to the sine of half the true angle between the objects; the double of which is the angle required.

43. Of the effect of the refractions of the rays in a glass mirror whose two surfaces are parallel.

Let AL and BN in figure 10 be the two parallel surfaces of a glass mirror. Q the focus of incident rays. QC a ray incident on the first surface at C. Through Q draw $QABS$ perpendicular to both surfaces, cutting

ting the firſt ſurface in *A*, the ſecond in *B*; produce *AQ* backwards to *R*, ſo that *AR* may be to *AQ* as the ſine of incidence to the ſine of refraction, or as *m* to *n*, and *R* will be the focus after refraction at *C* *. Draw *CD* the refracted ray meeting the ſecond ſurface in *D*. In the perpendicular *RS*, take *BS* equal to *BR*, and *S* will be the focus of rays reflected from the ſecond ſurface at *D* †. Draw *DE* the reflected ray meeting the firſt ſurface in *E*. Take *AT* to *AS* as *n* to *m*, and *T* will be the focus of emergent rays after their laſt refraction at *E* *; that is, *T* will be the place of the laſt image formed by two refractions and one reflection. Take *AV* equal to *AQ*, and *V* will be the place of the image formed by the reflection of the firſt ſurface only, and *TV* will be the diſtance between theſe two images. I ſay that *TV* will be to *AB* the thickneſs of the glaſs, as twice the ſine of refraction to the ſine of incidence.

44. Produce *TE* to *M* ſo that *EM* may be the emergent ray; through *D* draw the perpendicular *DH*, and through *E* and *C* the perpendiculars *be* and *dc*. Produce *QC*

* *Smith's* Optics, Art. 223. † Ibid. Art. 202.

till

till it meets TE in F, and through F draw FG perpendicular to QT. Now $dCD = CDH$ * $= HDE$ † $= DEb$ *; but DC and DE may be considered as rays incident on AL ‡; and because the angles of incidence dCD and DEb are equal, therefore their angles of refraction cCQ and eEM are equal; but $cCQ = GQF$ *, and $eEM = GTF$ *, therefore the triangle QFT is isosceles and $QG = GT$. Now $AT = GT + AG$ and $AV = AQ = GQ - AG = GT - AG$; whence $AT - AV$ or $TV = \overline{GT + AG} - \overline{GT - AG} = 2 \times AG$. Now the triangles CFH and CQA, as also CDH and CRA are respectively similar: hence HF is to AQ, also HD to AR, as CH to CA; therefore HF is to HD as AQ to AR ||; that is, AG is to AB as AQ to AR, or as n to m; consequently $2 \times AG$ or TV is to AB as $2n$ to m.

45. *Cor.* 1. If m be to n as 3 to 2; then $TV = \frac{4}{3}AB$ and $AG = \frac{2}{3}AB$ and $BG = \frac{1}{3}AB$.

46. *Cor.* 2. The last emergent ray proceeds exactly as if the reflection had been made without refraction, by a plane FG

* 29 El. I. † *Smith*'s Optics, Art. 8. ‡ Ibid. Art. 11. || 11 and 16 El. V.

parallel

parallel to *BN* the back ſurface, and diſtant from it $\frac{1}{3}$ of the thickneſs of the glaſs.

47. *Scholium.* The index-glaſs ſhould be ſo fixed on the index, that the center on which the index turns may paſs through the line *FG*; that is, be diſtant from the back ſurface $\frac{1}{3}$ of the thickneſs of the glaſs, and be within it.

48. We ſhall here juſt mention the conſequences that would follow from ſuppoſing the two ſurfaces of each glaſs mirror inclined to one another in a given angle: and we will conſider firſt the effect of the refractions of the index-glaſs only.

49. When there is no refraction, or when the two ſurfaces of the index-glaſs are parallel, then the laſt image is removed from the object by twice the inclination of the reflecting ſurfaces of the two ſpeculums, as we have ſhown before. But if the outer ſurface of the index-glaſs is inclined to the inner ſurface, the place of that image will then be changed by the refractions at the outer ſurface of the index-glaſs. The laſt image will in this caſe be formed by two refractions and one reflection at the index-

E glaſs:

glaſs: and its angular diſtance from the object will be increaſed, if the refracting angle of the index-glaſs be turned the ſame way with the angle which the ſpeculums make with each other; and decreaſed if the refracting angle be turned the contrary way. This angular change in the place of the laſt image on account of refractions we will call *the angle of deviation*.

50. Now was the angle of deviation always the ſame in every poſition of the index-glaſs, the refractions then would occaſion no error in the obſerved angle. For in this caſe the image, formed both by refraction and reflection, keeping one conſtant diſtance from that which would be formed by reflection only, will have the ſame angular motion, *viz*. double that of the index-glaſs. Therefore if any diſtant object and its laſt image, thus formed, be made to coincide when the index ſtands at the beginning of the diviſions; the index will then always ſhow the angular diſtance of this laſt image from the object, and conſequently the angular diſtance of any other object, which is ſeen *along with* that laſt image; juſt as when there is no refraction. All the difference

will be, that the reflecting ſurface of the index-glaſs will not ſtand exactly parallel to the horizon-glaſs when the object and it's image are made to coincide, in order to adjuſt the inſtrument.

51. When the rays fall ſo nearly perpendicular on the index-glaſs, that the angles of incidence and refraction may be accounted to have the given ratio of their ſines; then the angle of deviation is to the angle of inclination of the two ſurfaces of the index-glaſs, as double the difference between the ſines of incidence and refraction is to the leſſer of thoſe ſines. Hence the angle of deviation is nearly equal to the angle of the inclination of thoſe ſurfaces.

52. But as thoſe angles are not accurately as their ſines, ſo the angle of deviation is not exactly what that rule gives it; nor the ſame in all poſitions of the index-glaſs. It is always greater than what the rule gives; but it is leaſt of all when the ray within the glaſs falls perpendicularly on the reflecting ſurface, ſo that the firſt incident and laſt emergent rays coincide. This will be the caſe when *the index is in ſuch a poſition, that the horizon-glaſs may be ſeen*

by its own reflection. The incident ray in this case falls obliquely on the outer surface of the index-glass, and on that side of its perpendicular which is contrary to the refracting angle. We shall call this ray, *the ray of least deviation.*

53. The position of the index when the deviation is least being determined as before, the deviation will continually increase, as the distance of the index from this position increases on either side. If equal angular distances of the index from this position be taken on contrary sides, the angles of deviation in these two positions of the index will be nearly equal to each other, but not exactly so. That position of the index, in which the ray incident on the outer surface falls on the same side of the ray of least deviation with the perpendicular to that surface, will have the greatest angle of deviation.

54. All these consequences follow from the common properties of the prism, and need not be here particularly demonstrated. If a prism, whose refracting angle is about ten degrees, be properly placed before an index-glass whose surfaces are parallel, all these

these consequences may be shown experimentally *.

55. We have hitherto taken no notice of the refractions of the horizon-glass: it is because these refractions can never affect the observed angle. For as the horizon-glass is fixed, the rays always fall upon it with the same degree of obliquity. Hence the change in the place of the image, on account of the refractions of this glass, is constantly the same. The refractions of this glass may change a little the position of the axis of vision, but cannot alter the observed angle; not though the upper part of this glass be cut off, so that the object to be seen by direct vision may be unrefracted †, while

* It may be necessary to measure practically the angle of the prism. The following method is a very accurate one. Place the prism in a vertical position. Then, standing before the angle to be measured, place two candles or other proper objects, so that the image of one may be seen by reflection from one of the sides, and the image of the other be seen by reflection from the other of the two sides that form the refracting angle. Move these two objects till their reflected images coincide. Measure then the distances of these objects from each other and from the angle of the prism; from these distances compute the angle which the two objects subtend at the prism: half this is the angle sought. It may be necessary to cover the back of the prism, that the light refracted through the other two angles may not mix with that reflected from the sides.

† The angle of deviation from refraction only, is half that caused by refraction and reflection together as before.

the

the refractions affect the other object. This will only alter the position of the glasses in the adjustment as before. In like manner if the surfaces of the dark glasses be not parallel, their refractions will not affect the observations, provided the instrument be adjusted by the sun, or some object whose reflected image is seen through them.

56. If the inclination of the two surfaces of any of the glasses be considerable, the object will appear coloured, or at least indistinct; and if the instrument be turned about (in its own plane) the place of the reflected image will now be changed; because the angle of incidence, and therefore the effect of the refractions upon that image, will be changed. The multiplicity of the images may also create confusion. Upon these accounts those surfaces ought to be made parallel. But if this was not the case, we may see that it is impossible to remedy *perfectly* the other errors from refraction, notwithstanding what has been said by some persons of distinguished learning *.

* See the *American Transactions*, Vol. I. printed at *Philadelphia*, 1771, Appendix, page 21.

THE END.

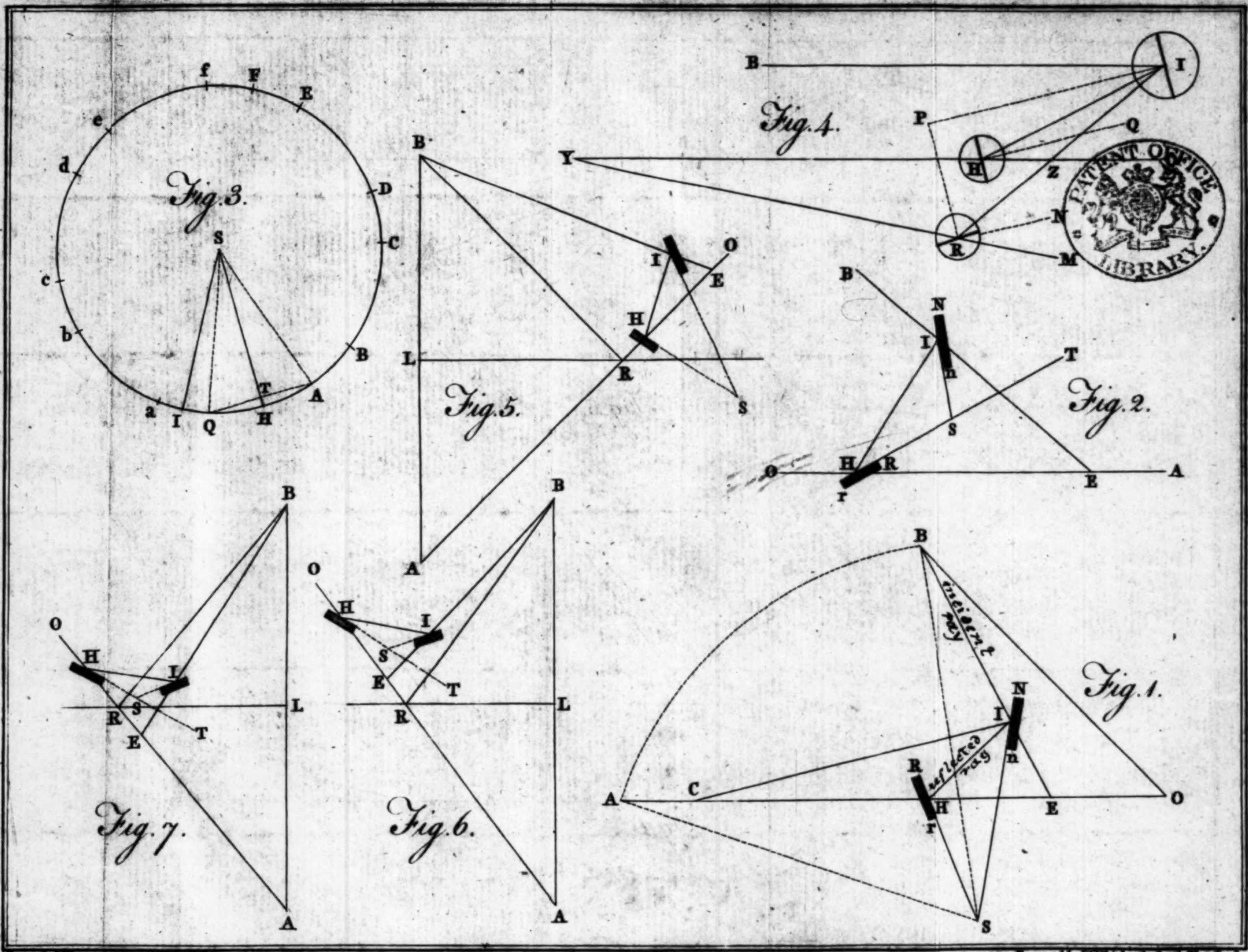

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Fig. 8.

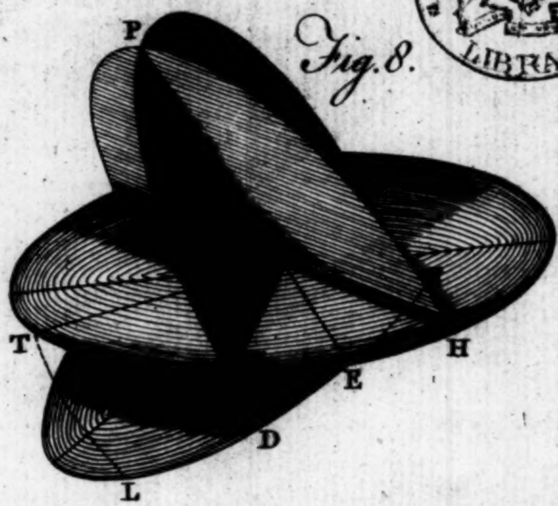


Fig. 10.

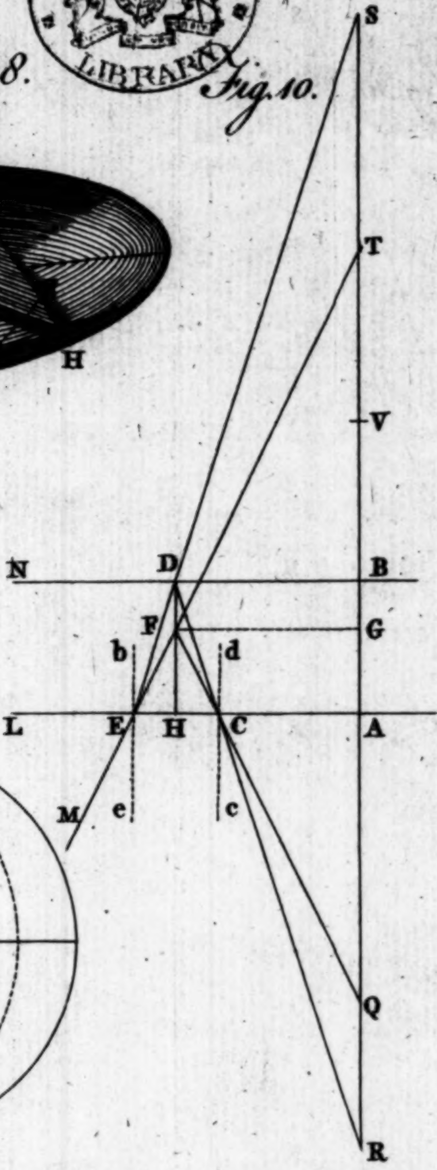
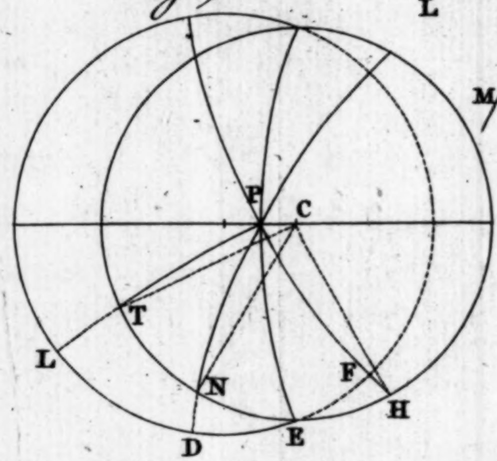


Fig. 9.



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